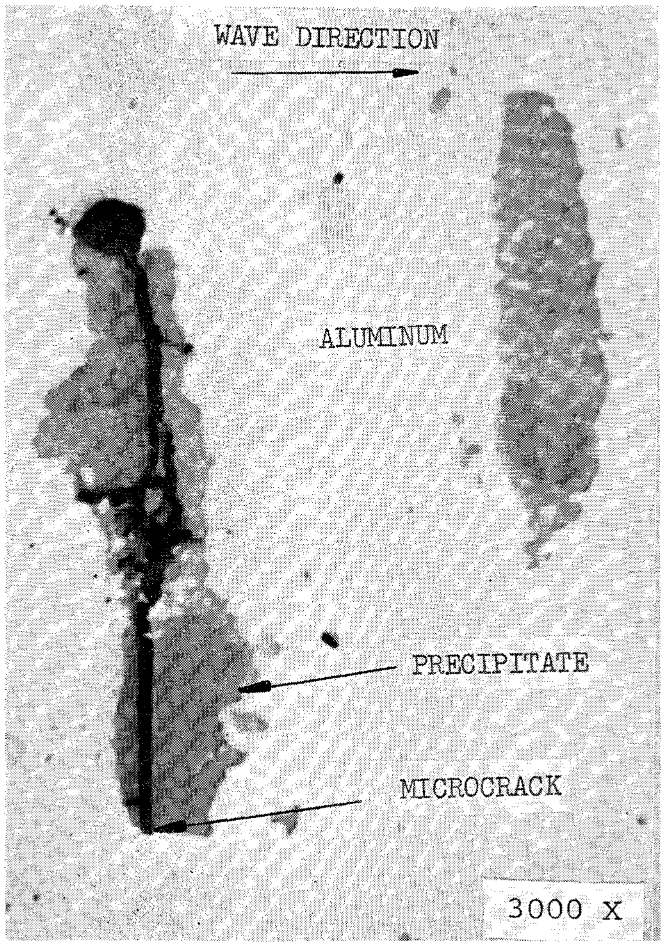

Fig. 4 Photograph of micro-crack in particle embedded in aluminum matrix.

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Vortices in Separated Flows

ALLEN H. WHITEHEAD JR.*
NASA Langley Research Center, Hampton, Va.

SEVERAL experimentalists[1–3] have shown that vortices existing in separated flows can result in a variation in the properties in the spanwise direction. The experimental evidence of Ginoux[1] and Roshko and Thomke[2] indicated the presence of vortices in the vicinity and downstream of reattachment. The experiment of Reding et al.[3] on a cone-cylinder-flare configuration with local transonic conditions verified the existence of vortices and a strong circulatory flow within a transitional-separated region; the resulting separation and reattachment points varied widely around the circumference. These observed variations have led Reding et al.[3] to conclude that the assumption of axisymmetric separated flow is unrealistic. Evidence of symmetric separation is presented in the present Note which indicates that Reding's conclusion is not universally valid. The role of vortices in separated flows is also examined.

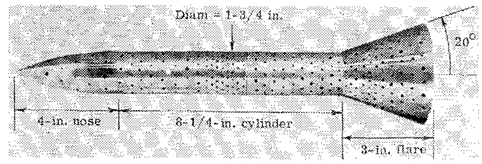

Fig. 1 Model dimensions and oil-dot pattern prior to run. Dimensions in inches.

An experimental study was conducted at Mach 6.8 in the Langley 11-in. tunnel at a unit Reynolds number of 0.33×10^6 per in. on a model similar to that employed by Reding, but with the conical nose replaced by an ogive nose. The schlieren pictures indicate that transition occurs within the separated region just downstream of the cylinder-flare juncture. The transition position and separated-flow characteristics in the present tests are compatible with previous results on geometrically similar models run in the same facility.[4] Figure 1 shows the model dimensions and illustrates the oil-dot technique employed in the study; the dots are composed of a mixture of oil and lampblack, and have been placed on the model in a systematic manner with a higher density of dots in the anticipated areas of separation and reattachment.

Three views of the oil-flow photographs from this investigation are shown in Fig. 2. The separation and reattachment regions as indicated by the oil flow show that the separated flow is symmetrical about this configuration. There is no indication of the large circumferential circulation in the plane of the body symmetry reported by Reding. Similar symmetric characteristics were observed in oil flow studies of turbulent separation over a cylinder and 40° flare studied by Polak at Mach 5, 6, and 7.[5] Polak's oil flow results were part of a larger study to examine separation-length characteristics at high Reynolds numbers.[6]

The asymmetric separation observed by Reding et al.[3] at zero angle of attack could be atrributed to several factors such as 1) the general unsteadiness that is characteristic of transonic flows and 2) the occurrence of a transitional-separated boundary layer. Chapman[7] has shown that for flat-plate transitional separation, the transition position is sensitive to a Reynolds number variation. As a result, the characteristics of the transitional-separated region are likewise sensitive to small changes in surface and flowfield properties. In contrast, laminar, and turbulent separations are known to be far

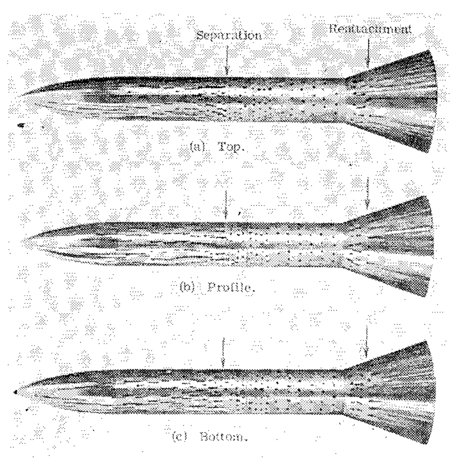

Fig. 2 Oil-flow pictures at $\alpha = 0°$; $M_\infty = 6.8$; R_∞/in. = 0.33×10^6.

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* Aerospace Engineer, Applied Fluid Mechanics Section, Aero-Physics Division. Associate AIAA.

less Reynolds number dependent. In view of this reasoning, it is not surprising that for transitional separation at transonic conditions, the transition location can vary around the circumference of a symmetric configuration, thereby producing an asymmetric flow. Within the resulting asymmetric separated region, rather large vortices are created that transport fluid from one region to another around the circumference. When a symmetric configuration is brought to an angle of attack, crossflow introduces nonsymmetric flow separation and vortex motion problably results regardless of the type of separation or the freestream Mach number.

Similar large-scale vortices were observed by Whitehead[8] at Mach 6 within an irregularly shaped separated region on a delta wing at zero angle of attack with a 40° trailing edge flap. As discussed in Ref. 8 the shape of the separated region and the nature of the subsequent vortex formation is similarly dependent on the location of transition across the span. Further confirming this conclusion is the change in the character of the separated region when the local Reynolds number is increased; a more regular separation appears in which the vortices are considerably reduced in size and are confined to the outboard region of the hinge line. The large-scale vortices observed on Reding's axisymmetric model and on the delta wing of Ref. 8 are responsible for transporting a portion of the flow out of the separated region. Thus the classical Chapman-Korst separated-flow model, which assumes that fluid can depart the separated region only by passing through the mixing region adjacent to the dividing streamline, is not applicable in these cases.

In contrast to these large-scale vortices are the small-scale vortices observed by Roshko and Thomke[2] in the reattachment region on an axisymmetric rearward-facing step, and by Ginoux[1,9] on a two-dimensional rearward facing step. The effect of these vortices on the reattachment flow and on the mass transferred out of the separated region has not been fully determined, but detailed pressure distributions in the turbulent reattachment region by Roshko and Thomke[2] indicate only a small effect on the static pressure distribution (less than 2% circumferential variation). Furthermore, while Roshko and Thomke's oil-flow results give evidence of the small-scale vortices in the reattachment region, there is no evidence of the circumferential communication which was observed by Reding et al.[3] Ginoux[9] states that the source of these vortices arises from small irregularities in the leading edge. In his studies, he observed regular spanwise variations in pitot measurements within the reattached boundary layer. The magnitude of the resultant spanwise heating variations was strongly dependent on the type of separation; only a 7% variation was recorded for turbulent reattachment compared to over a 100% variation at reattachment of a transitional-separated flow. In a two-dimensional, turbulent-separation study by Sterrett et al.[10] at Mach 6, oil-flow traces of vortices shed from spherical roughness elements appeared in the vicinity and downstream of reattachment on a 40° wedge. Only small spanwise variations were observed in the heating caused by these vortices.

The assertion by Reding et al.[3] that the presence of vortices in separated flows precludes the existence of symmetric flows does not consider the origin or size of the vortices. The small-scale vortices can be considered small perturbations on an existing flowfield and, as such, have a smaller effect on the flowfield and on surface properties than the large-scale vortices that can alter the flowfield significantly. Transitional separation apparently enhances the formation of large-scale vortices, and maximizes the amplitude of the spanwise variations resulting from any small-scale vortices. The present evidence has shown, however, that a symmetric flow field can exist over symmetric configurations for both transitional and turbulent separation.

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Stability of Dynamic Systems Subjected to Nonconservative and Harmonic Forces

F. C. L. Fu*
University of California, San Diego, La Jolla, Calif.
AND
S. Nemat-Nasser†
Northwestern University, Evanston, Ill.

1. Introduction

CONSIDER a dynamic system represented by a set of linear ordinary differential equations with periodic coefficients as follows:

$$\ddot{x}_\alpha + \Omega_{\alpha\beta}x_\beta + \epsilon\varphi_{\alpha\beta}(t)x_\beta = 0, \quad \alpha,\beta = 1, 2, \ldots, N \qquad (1)$$

where $\Omega_{\alpha\beta} = 0$ if $\alpha \neq \beta$ and $\Omega_{\alpha\beta} > 0$ if $\alpha = \beta$, ϵ is an amplitude parameter, $\varphi_{\alpha\beta}$ are symmetric periodic functions of a period T with zero mean values, and where the summation convention on repeated indices is used and will also be employed in the sequel. The stability of this system under a set of rather weak assumptions regarding the periodic functions $\varphi_{\alpha\beta}$ has been extensively studied by a number of investigators, see Cesari,[1] Mettler,[2] Hale,[3–5] Gambill,[5–7] and Cesari and Bailey[8]; for further references see Cesari.[1] These investigations are based on a method originally proposed by Cesari where, by means of a convergent successive approximation technique, explicit relations are obtained for the so-called characteristic exponents of the system.

When a dynamic system is subjected to circulatory, gyroscopic, and other nonconservative forces, Eq. (1) must be

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* Graduate Student.

† Professor.